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Title

HOMOGENEOUS MARKOV PROCESSES ON DISCRETE TIME QUEUES TO CONTINUOUS TIME

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ABSTRACT:

Discrete time analysis queues have always been popular with engineers who are very keen on obtaining numerical values out of their analyses for the sake of experimentation and design. As telecommunication systems are based more on digital technology these days than analog the need to use discrete time analysis for queues has become more important. Besides, we find that several queues which are difficult to analyze by the continuous time approach are sometimes easier to analyze using discrete time method. Of course, there are some queueing problems which are easier to analyze using continuous time approach instead of discrete time. We discuss, in this paper, to transfer the discrete time results to Markov processes in continuous time.

KEYWORD: - Discrete time – Queues, Queueing System, Markov process, Optimization, Poisson distribution, Exponential distribution.

INTRODUCTION:

In *continuous-state* models the state space *X* is a continuum consisting of all *n*-dimensional vectors of real (or sometimes complex) numbers. Normally, *X* is finite dimensional (i.e., *n* is a finite number), although there are some exceptions where *X* is infinite-dimensional. In *discrete-state* models the state space is a discrete set. In this case, a typical sample path is a piecewise constant function, since state variables are only permitted to jump at discrete points in time from one discrete state value to another. Naturally, there are many situations in which a *hybrid* model may be appropriate, that is, some state variables are discrete and some are continuou**s.**

Now define $T_0 = 0$ and let $(T_n : n \in N)$ denote a sequence of positive real-valued random variables with $T_{n+1} > T_n$ for all $n \in N_0$ and $T_n \to \infty$ as $n \to \infty$ Further, let E denote a countable state space and $(X_n : n \in N_0)$ a sequence of E-valued random variables. A process $\Upsilon = Y_t : t \in R_0^+$ in continuous time with

$$Y_t = X_n \quad for \;\; T_n \le t < T_{n+1}$$

is called a pure jump process. The variable $H_n = T_{n+1} - T_n$ is called the nth holding time of the process Υ. If further $\chi = X_n : n \in N_0$ is a Markov chain with transition matrix $P = p_{ij}, ij \in E$ and the variables H_n are independent and distributed exponentially with parameter λX_n only depending on the state X_n, then Υ is called homogeneous **Markov process** with discrete **state space** *E*. The chain χ is called the **embedded Markov chain** of Υ.

As a technical assumption we always agree upon the condition $\lambda = \sup \lambda_i : i \in E < \infty$ i.e. the parameters for the exponential holding times shall be bounded.

An immediate consequence of the definition is that the paths of a Markov process are step functions. The lengths of the holding times are almost certainly strictly positive, since exponential distributions are zero with probability zero.

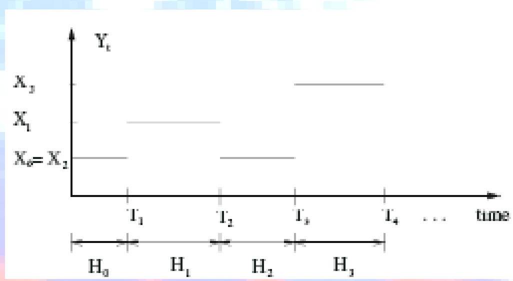


Figure 1.1. Typical path of a Markov process with discrete state space

EXAMPLE 1.1 POISSON PROCESS

Define $X_n = n$ deterministically. Then $\chi = X_n : n \in N_0$ is a Markov chain with state space $E = N_0$ and transition probabilities $p_{n,n+1} = 1$ for all $n \in N_0$. Let the holding times H_n be distributed exponentially with identical parameter $\lambda > 0$. Then the resulting process γ as defined in the above definition is a Markov process with state space N0. It is called **Poisson process** with **intensity** (also: rate or parameter) λ.

Next we want to prove a property similar to the Markov property for Markov chains in discrete time. To this aim, we need to show the **memoryless property** for the exponential distribution, which is the analogue to the memoryless property for geometric distributions in discrete time.

LEMMA 1.1 Let H denote a random variable having an exponential distribution with parameter λ. Then the memoryless property

$$P(H > t + s | H > s) = P(H > t)$$

holds for all time durations s, t > 0.

PROOF: We immediately check

$$P(H > t + s | H > s) = \frac{P(H > t + s | H > s)}{P(H > s)} = \frac{P(H > t + s)}{P(H > s)}$$
$$= \frac{e^{-\lambda - t+s}}{e^{-\lambda - s}} = e^{-\lambda - t} = P\ \ H > t$$

which holds for all s, t > 0.

THEOREM 1.1 Let Y denote a Markov process with discrete state space E. Then the **Markov** ***property***

$$P(Y_t - j | Y_u : u \le s) = P(Y_t - j | Ys)$$

holds for all times s < t and states j ∈ E.

PROOF: Denote the state at time s by $Y_s = i$. Because of the memoryless property of the exponential holding times, the remaining time in state i is distributed exponentially with parameter λ i, no matter how long the preceding holding time has been. After the holding time in the present state elapses, the process changes to another state j according to the homogeneous Markov chain χ. Hence the probability for the next state being j is given by p_{ij}, independently of any state of the process before time s. Now another exponential holding time begins, and thus the past before time s will not have any influence on the future of the process γ.

Now in the discrete time case, for any two time instances $s < t$ the conditional probabilities $P(Y_t = j | Ys = i)$ shall be called the **transition probabilities** from time *s* to time *t*. We will now derive a recursion formula for the transition probabilities of a Markov process by conditioning on the number of jumps between time *s* and time *t*:

.**THEOREM 1.2** The transition probabilities of a Markov process γ are given by

$$P\left(Y_t = j | Ys = i\right) = \sum_{n=0}^{\infty} P_{ij}^{n}\left(s,t\right)$$

for all times s < t and states i, j ∈ E, with

$$P_{ij}^{0}\left(s,t\right) = \delta_{ij}.e^{-\lambda_i.\left(t-s\right)}$$

and recursively

$$P_{ij}^{n+1}\left(s,t\right) = \int_s^t e^{-\lambda_i.u}\lambda_i \sum_{k\in E} P_{ik} P_{kj}^{n}\left(u,t\right) du$$

for all $n \in N_0$.

PROOF: The above representation follows immediately by conditioning on the number of jumps in]s, t]. The expressions $P_{ij}^{n}\left(s,t\right)$ represent the conditional probabilities that $Y_t = j$ and there are n jumps in]s, t] given that $Y_t = i$ In the recursion formula the integral comprises all times *u* of a possible first jump along with the **Lebesgue** density $e^{-\lambda_i.u}\lambda_i$ of this event, after which the probability of *n* remaining jumps reaching state *j* at time *t* is given by $\sum_{k\in E} P_{ik} P_{kj}^{n}\left(u,t\right)$

For every two time instances *s* < *t*, define the **transition probability matrix** *P*(*s*, *t*) from time *s* to time *t* by its entries

$$P_{ij}\left(s,t\right) = P(Y_t = j | Ys = i)$$

Using the recursion formula, it is shown by induction on n that the conditional probabilities $P_{ij}^{n}\left(s,t\right)$ are homogeneous in time, i.e. they satisfy

$$P_{ij}^{n}\left(s,t\right) = P_{ij}^{n}\left(0,t-s\right)$$

for all *s* < *t*. Thus we can from now on restrict the analysis to the transition probability matrices

$$P(t) = P(0,t)$$

with $t \geq 0$. With this notation the Markov property yields the **Chapman– Kolmogorov equations**

$$P(s+t) = P(s)P(t)$$

for all time durations $s,\ t \geq 0$. Thus the family $\{ P(t) : t \geq 0 \}$ of transition probability matrices forms a semi–group under the composition of matrix multiplication. In particular, we obtain for the neutral element of this semi–group $P(0) = I_E := (\delta_{ij})_{i,j \in E}$ with $\delta_{ij} = 1$ for $i - j$ and zero otherwise.

In order to derive a simpler expression for the transition probability matrices, we need to introduce another concept, which will be called the **generator matrix**. This is defined as the matrix $(g_{ij})_{i j \in E}$ on E with entries

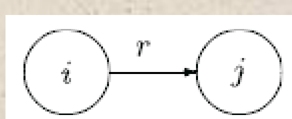
$$g_{ij} = \begin{cases} -\lambda_i \cdot (1 - p_{ij}), & i = j \\ \lambda_i \cdot p_{ij}, & i \neq j \end{cases}$$

for all states $i, j \in E$. In particular, the relation

$$g_{ij} = -\sum_{j \neq i} g_{ij} \qquad (1.1)$$

holds for all $i \in E$.

The $(i,\ j)$th entry of the generator G is called the **infinitesimal transition rate** from state i to state j. Using these, we can illustrate the dynamics of a Markov process in a directed graph where the nodes represent the states and an edge



means that $g_{ij} = r > 0$. Such a graph is called a **state transition graph** of the Markov process. With the convention $p_{ii} = 0$ the state transition graph uniquely determines the Markov process.

THEOREM 1.3 The transition probabilities Pij(t) of a Markov process satisfy the systems

$$\frac{dP_{ij}\ t}{dt} - \sum_{k\in E} P_{ik}\ t\ g_{kj} - \sum_{k\in E} g_{ik} P_{kj}\ t$$

of differential equations. These are called the **Kolmogorov forward and backward equations**.

PROOF: From the representation in theorem 1.2, it follows by induction on the number of jumps that all restricted probabilities $P^{(n)}(t)$ are Lebesgue integrable with respect to t over finite intervals. Since the sum of all $P_{ij}^{\ n}\ t$ is a probability and thus bounded, we conclude by majorized convergence that also P (t) is Lebesgue integrable with respect to t over finite intervals.

Now we can state the recursion

$$P_{ij}\ t\ = e^{-\lambda_i . t} . \delta_{ij} + \int_0^t e^{-\lambda_i . s}\ \lambda_i \sum_{k\in E} p_{ik} P_{kj}\ t-s\ ds$$

which results from conditioning on the time s of the first jump from state i . We obtain further

$$P_{ij}\ t\ = e^{-\lambda_i . t} . \left(\delta_{ij} + \int_0^t e^{+\lambda_i . u}\ \lambda_i \sum_{k\in E} p_{ik} P_{kj}\ u\ du \right)$$

by substituting u = t − s in the integral. Since $\sum_{k\in E} p_{ik} = 1$ is bounded, we conclude that P(t) is continuous in t. Further, we can differentiate P(t) as given in the recursion and obtain

$$\frac{dP_{ij}\ t}{dt} = -\lambda_i e^{-\lambda_i . t}\ \ \delta_{ij} + \int_0^t f\ \ u\ du\ + e^{-\lambda_i . t} . f\ \ t$$

with f denoting the integrand function. This means nothing else than

$$\frac{dP_{ij}\ \ t}{dt} = -\lambda_i P_{ij}\ \ t\ \ + \lambda_i \sum_{k \in E} p_{ik} P_{kj}\ \ t$$

$$= -\lambda_i\ \ 1 - p_{ii}\ \ .P_{ij}\ \ t\ \ + \sum_{k \neq i} g_{ik} P_{kj}\ \ t$$

and thus proves the backward equations. For the forward equations, one only needs to use the Chapman–Kolmogorov equations and apply the backward equations in

$$\frac{dP_{ij}\ \ t}{dt} = \lim_{h \to 0} \frac{P_{ij}\ \ t+h\ \ - P_{ij}\ \ t}{h} = \lim_{h \to 0} P_{ik}\ \ t\ \ \frac{P_{kj}\ \ h\ \ - \delta_{kj}}{h}$$

$$= \sum_{k \in E} P_{ik}\ \ t\ \ \lim_{h \to 0} \frac{P_{kj}\ \ h\ \ \ P_{kj}\ \ 0}{h} \sum_{k \in E} P_{ik}\ \ t\ \ g_{kj}$$

which holds for all i, j ∈ E.

CONCLUSION:

Above theorem transfers the discrete time results to Markov processes in continuous time. This was the complete proof.

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